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String bits without doubling

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Abstract: We construct a string bit model in the pp-wave background in which fermion doubling produces the correct spectrum of string states.

Keywords: Lattice Quantum Field Theory, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

The type-IIB superstring theory in the pp-wave background with Ramond-Ramond flux [1, 2] has drawn considerable attention recently, mainly because it arises in a certain limit of AdS/CFT correspondence [3, 4, 5] and describes a particular set of operators in the dual super-Yang-Mills (SYM) theory [5]. The relationship between string states and field-theory operators, a cornerstone feature of the gauge/string theory duality [6], is remarkably simple and explicit in the pp-wave limit. The operators in some sense correspond to discretized strings which are built out of a finite number of partons [5]. The parton picture of the string, and also the fact that string theory in the pp-wave is most easily quantized in the light-cone gauge, is suggestive of the use of the string bit models which were proposed a long time ago to describe strings of partons in the light-cone frame [7]. The key observation is that the length of the string in the light-cone gauge is proportional to light-cone momentum P^+ [8]. Therefore it can be interpreted as an ordered array of partons, each of which carries a fixed small portion of the P^+. The continuous string is recovered in the limit when the number of partons becomes infinite.

In the context of the pp-wave string partons are elementary fields of the SYM theory and the string hamiltonian arises from their perturbative interactions [5]. The string bit model in the pp-wave background was introduced in [9, 10] and gives an elegant unified description of the multi-string Hilbert space and of the string interactions. The string interactions correspond to $1/N$ corrections in the SYM theory, and it was proposed that the string bit model can describe operator mixing in the SYM theory non-perturbatively in $1/N$ [10, 11, 12] (see [13, 14] for the discussion of operator mixing on the field-theory side).

As was noticed in [15], world-sheet fermions of the string bit model potentially cause a problem because of fermion doubling. The problem is actually not specific to the pp-wave string and can arise in any supersymmetric string bit model. The fermion doubling implies the appearance of spurious low-energy modes of fermions in lattice field theory [16]. The wave functions of these modes are sign-alternating with the period of two lattice spacings, i.e. they have a momentum of the order of the lattice cutoff. Their energy is nevertheless finite. In the context of the string bit model, doublers threaten to destroy the correct string spectrum in the continuum limit and should be eliminated before the continuum limit is taken.

We advocate an alternative approach to the doubling problem. Instead of removing the doublers, we propose to reduce the fermion content of the string bit model by half and to regard doublers as physical states. Our approach closely follows one of the well-known methods to tackle fermion doubling in lattice field theories [17, 18]. The fermion doubling will automatically recover the correct string spectrum in the continuum limit. An important point to note here is that doubling produces a pair of identical fermions, and indeed there are two world-sheet fermions in the type-IIB string theory with the same quantum numbers.

The outline of the paper is as follows. In section two we introduce a new string-bit hamiltonian, in section three we propose new supersymmetry generators, in section four we investigate their algebra, and in section five, finally, we present some conclusions.

2. The string-bit hamiltonian

The light-cone hamiltonian for the IIB Green-Schwarz superstring in the pp-wave background is given by [1][1]

$$H_{\text{continuum}} = \frac{1}{2}\int d\sigma \left(p^2 + x'^2 + x^2 + i\theta\theta' - i\tilde{\theta}\tilde{\theta}' + 2i\theta\Pi\tilde{\theta}\right). \tag{2.1}$$

The essential ingredient of the string-bit model is a discretized version of this hamiltonian. The main advantage of the discretization is the possibility to describe string interactions in a combinatorial way. The Fock space of a discretize string naturally includes multi-string states, and consequently string interactions can be introduced by just adding extra terms to the hamiltonian [9]. Such a simple description of interactions is impossible for the continuous string. Our main goal, however, is to address the doubling problem, which arises before the Fock space is extended to include multiple strings, and we shall concentrate on the Hilbert space of a single string in what follows.

The string bit hamiltonian of [9] is the straightforward discretization of (2.1):

$$H_{\text{string bit}} = \frac{a}{2}\sum_{n=1}^{J}\left[p_n^2 + x_n^2 + \frac{1}{a^2}(x_{n+1}-x_n)^2 + \frac{i}{a}\,\theta_n\theta_{n+1} - \frac{i}{a}\,\tilde{\theta}_n\tilde{\theta}_{n+1} - 2i\tilde{\theta}_n\Pi\theta_n\right]. \tag{2.2}$$

To be more precise, this hamiltonian is the gauge-fixed version of the string bit model. We believe, however, that a gauge-invariant generalization of our approach, and an inclusion of multi-string states, should not cause any serious problems. In the above expression the

[1]We use the notations and conventions of [19] for SO(8) spinors and Dirac matrices. The word-sheet fermions are in the $\mathbf{8_s}$ of SO(8). The mass matrix $\Pi_{ab} = \gamma^1_{a\dot{a}}\gamma^2_{\dot{a}c}\gamma^3_{c\dot{b}}\gamma^4_{\dot{b}b}$ is symmetric and traceless, and explicitly breaks SO(8) to SO(4) $\times$ SO(4).

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inverse lattice spacing $1/a$ should be identified with the 't Hooft coupling of the dual SYM theory [9], and the phase space coordinates are understood to obey the following canonical commutation relations

$$[p_n^i, x_m^j] = -\frac{i}{a}\,\delta^{ij}\delta_{nm}\,, \qquad \{\theta_n^a, \theta_m^b\} = \frac{1}{a}\,\delta^{ab}\delta_{nm}\,, \qquad \{\tilde\theta_n^a, \tilde\theta_m^b\} = \frac{1}{a}\,\delta^{ab}\delta_{nm}\,. \tag{2.3}$$

Here $i = 1\dots 8$ and $a = 1\dots 8$ are indices of the vector and of one of the spinor representations of SO(8). The other spinor representation will be denoted by dotted indices.

A straightforward discretization of the supersymmetry generators yields:

$$\begin{aligned} Q_{\text{string bit}} &= a\sum_{n=1}^{J}\left[p_n^i\gamma^i\theta_n - x_n^i\gamma^i\Pi\tilde\theta_n + \frac{1}{a}(x_{n+1}^i - x_n^i)\gamma^i\theta_n\right], \\ \widetilde{Q}_{\text{string bit}} &= a\sum_{n=1}^{J}\left[p_n^i\gamma^i\tilde\theta_n + x_n^i\gamma^i\Pi\theta_n - \frac{1}{a}(x_{n+1}^i - x_n^i)\gamma^i\tilde\theta_n\right]. \end{aligned} \tag{2.4}$$

The hamiltonian and the supercharges are diagonalized by Fourier transforms given by

$$x_n = \frac{1}{\sqrt{J}}\sum_{p=-J/2}^{J/2-1} x_p e^{2\pi i p n/J}\,, \qquad p_n = \frac{1}{\sqrt{J}}\sum_{p=-J/2}^{J/2-1} p_p e^{2\pi i p n/J}\,, \tag{2.5}$$

$$\theta_n = \frac{1}{\sqrt{J}}\sum_{p=-J/2}^{J/2-1} \theta_p e^{2\pi i p n/J}\,, \qquad \tilde\theta_n = \frac{1}{\sqrt{J}}\sum_{p=-J/2}^{J/2-1} \tilde\theta_p e^{2\pi i p n/J}\,, \tag{2.6}$$

which leads to a hamiltonian of the form

$$H = \frac{a}{2}\sum_{p=-J/2}^{J/2-1}\left[p_p p_{-p} + x_p x_{-p} + \frac{4}{a^2}x_p x_{-p}\sin^2\frac{p\pi}{J} + \frac{1}{a}\left(\theta_p\theta_{-p} - \tilde\theta_p\tilde\theta_{-p}\right)\sin\frac{2p\pi}{J} - 2i\tilde\theta_p\Pi\theta_{-p}\right]. \tag{2.7}$$

It is now easy to see that the kinetic energy of fermions has two zeros, at $p = 0$ and at $p = J/2$, in each Brillouin zone. As a consequence there are twice as many light fermion states as there should be in the continuum limit. The bosons do not suffer from this problem, since modes with $p \sim J/2$ have energies of order of the cutoff and decouple in the continuum limit. The fermions states with momentum close to $J/2$ are potentially dangerous, since their energy is finite in the continuum limit and they may mix with physical states when string interactions are taken into account. In addition, as shown in [15], the variation of H under the supersymmetry transformations fails to vanish.

We propose a mild modification of the string bit model which is free from the doubling problem. Actually, instead of viewing it as a problem, we shall take advantage of the fermion doubling. The key observation is that the spectrum of the type-IIB superstring contains two fermions transforming in the same way. This suggests that the right spectrum can emerge as the result of the doubling of a single fermion species. The idea is then to start out with half the number of fermions in the discretized case and let the fermion doubling provide for the missing states in the continuum limit.

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Since the momenta of the doublers lie close to the boundary of the Brillouin zone, their wave functions in the coordinate space are approximately sign alternating with the period of two lattice spacings. We can couple doublers to the normal low-momentum modes by introducing the sign factor $(-1)^n$. A hamiltonian that achieves this goal is obtained from (2.2) by omitting the $\tilde{\theta}$ and introducing the staggered mass term:[2]

$$H = \frac{a}{2}\sum_{n=1}^{J}\left[p_n^2 + x_n^2 + \frac{1}{a^2}(x_{n+1}-x_n)^2 + \frac{i}{a}\,\theta_n\theta_{n+1} - i(-1)^n\theta_n\Pi\theta_{n+1}\right] \tag{2.8}$$

$$= \frac{a}{2}\sum_{p=-J/2}^{J/2-1}\left(p_p p_{-p} + x_p x_{-p} + \frac{4}{a^2}\,x_p x_{-p}\sin^2\frac{p\pi}{J} + \frac{1}{a}\,\theta_p\theta_{-p}\sin\frac{2p\pi}{J} + i\theta_{p+J/2}\Pi\theta_{-p}\cos\frac{2p\pi}{J}\right). \tag{2.9}$$

In the continuum limit, where $J \to \infty$ and $a \to 0$, we recover (2.1) if we make the identification

$$\tilde{\theta}_p = \theta_{p+J/2}\,. \tag{2.10}$$

There are two sets of low-energy modes: θ_p and $\tilde{\theta}_p$ with $p \ll J$. In the coordinate representation the physical fermions are linear combinations of lattice variables on two neighboring sites:

$$\begin{aligned}\theta_n^a &\to \frac{1}{2}(\theta_n^a + \theta_{n+1}^a)\\ \tilde{\theta}_n^a &\to \frac{1}{2}(-1)^n(\theta_n^a - \theta_{n+1}^a)\,.\end{aligned} \tag{2.11}$$

That is, θ_p (for finite p) corresponds to fluctuations around a constant θ_n, while $\tilde{\theta}_p$ corresponds to fluctuations around a staggered state where θ_n is alternating between the lattice sites.

3. The supersymmetry transformations

We must now turn to the subject of supersymmetry. The discretization of the supercharges and the definition of the supersymmetry transformations on the lattice is a very subtle procedure. For instance, the straightforward substitution of the fermion variables (2.11) into the supercharges (2.4) does not work. The supersymmetry transformations then mix low-energy degrees of freedom that survive the continuum limit with heavy lattice modes, which renders truncation of the superalgebra to the low-energy sector inconsistent. We propose to start with the original supersymmetry generators in (2.4) above, make replacements according to (2.11), and in addition replace bosonic variables by their symmetric combinations

$$p_n^i \to \frac{1}{2}\left(p_n^i + p_{n+1}^i\right), \qquad x_n^i \to \frac{1}{2}\left(x_n^i + x_{n+1}^i\right). \tag{3.1}$$

[2]Strictly speaking, we should assume that the number of sites in the lattice is even. Otherwise, $(-1)^n$ is not a periodic function on the lattice, and the mass term will contain a "defect" near $n = 0$. Its effect, however, should disappear in the continuum limit, and for this reason the difference between lattices with even and odd number of sites will not affect our main results.

In this way we get for the supercharges:

$$Q = \frac{a}{4}\sum_{n=1}^{J}\Big[(p_n^i + p_{n+1}^i)\gamma^i(\theta_n + \theta_{n+1}) - (-1)^n(x_n^i + x_{n+1}^i)\gamma^i\Pi(\theta_n - \theta_{n+1}) + \\ + \frac{1}{a}(x_{n+2}^i - x_n^i)\gamma^i(\theta_n + \theta_{n+1})\Big]\,, \tag{3.2}$$

and

$$\widetilde{Q} = \frac{a}{4}\sum_{n=1}^{J}\Big[(-1)^n(p_n^i + p_{n+1}^i)\gamma^i(\theta_n - \theta_{n+1}) +_n^i + x_{n+1}^i)\gamma^i\Pi(\theta_n + \theta_{n+1}) - \\ - \frac{1}{a}(-1)^n(x_{n+2}^i - x_n^i)\gamma^i(\theta_n - \theta_{n+1})\Big]\,. \tag{3.3}$$

As we shall now show, these operators have the desired properties. Straightforward calculations give the following supersymmetry transformations:

$$[Q, x_n^i] = -\frac{i}{4}\gamma^i(\theta_{n+1} + 2\theta_n + \theta_{n-1})\,, \tag{3.4}$$

$$[Q, p_n^i] = \frac{i}{4}(-1)^n\gamma^i\Pi(\theta_{n+1} - 2\theta_n + \theta_{n-1}) - \frac{i}{4a}\gamma^i(\theta_{n+1} + \theta_n - \theta_{n-1} - \theta_{n-2})\,, \tag{3.5}$$

$$\{Q, \theta_n\} = \frac{1}{4}(p_{n+1}^i + 2p_n^i + p_{n-1}^i)\gamma^i - \frac{1}{4}(-1)^n(x_{n+1}^i + 2x_n^i + x_{n-1}^i)\gamma^i\Pi - \\ - \frac{1}{4a}(x_{n+2}^i + x_{n+1}^i - x_n^i - x_{n-1}^i)\gamma^i\,, \tag{3.6}$$

and

$$[\widetilde{Q}, x_n^i] = \frac{i}{4}(-1)^n\gamma^i(\theta_{n+1} - 2\theta_n + \theta_{n-1})\,, \tag{3.7}$$

$$[\widetilde{Q}, p_n^i] = \frac{i}{4}\gamma^i\Pi(\theta_{n+1} + 2\theta_n + \theta_{n-1}) - \frac{i}{4a}(-1)^n\gamma^i(\theta_{n+1} - \theta_n - \theta_{n-1} + \theta_{n-2})\,, \tag{3.8}$$

$$\{\widetilde{Q}, \theta_n\} = \frac{1}{4}(-1)^n(p_{n+1}^i + 2p_n^i + p_{n-1}^i)\gamma^i + \frac{1}{4}(x_{n+1}^i + 2x_n^i + x_{n-1}^i)\gamma^i\Pi - \\ - \frac{1}{4a}(-1)^n(x_{n+2}^i + x_{n+1}^i - x_n^i - x_{n-1}^i)\gamma^i\,. \tag{3.9}$$

After Fourier transforming these become

$$[Q, x_p^i] = -i\cos^2\frac{\pi p}{J}\gamma^i\theta_p\,, \tag{3.10}$$

$$[Q, p_p^i] = -i\cos^2\frac{\pi p}{J}\gamma^i\Pi\theta_{p-J/2} + \frac{2}{a}\sin\frac{\pi p}{J}\cos^2\frac{\pi p}{J}e^{-i\pi p/J}\gamma^i\theta_p\,, \tag{3.11}$$

$$\{Q, \theta_p\} = \cos^2\frac{\pi p}{J}p_p^i\gamma^i - \sin^2\frac{\pi p}{J}x_{p+J/2}^i\gamma^i\Pi - \frac{2i}{a}\sin\frac{\pi p}{J}\cos^2\frac{\pi p}{J}e^{i\pi p/J}x_p^i\gamma^i\,, \tag{3.12}$$

and

$$[\widetilde{Q}, x_p^i] = -i\cos^2\frac{\pi p}{J}\gamma^i\theta_{p+J/2}\,, \tag{3.13}$$

$$[\widetilde{Q}, p_p^i] = i\cos^2\frac{\pi p}{J}\gamma^i\Pi\theta_p - \frac{2}{a}\sin\frac{\pi p}{J}\cos^2\frac{\pi p}{J}e^{-i\pi p/J}\gamma^i\theta_{p+J/2}\,, \tag{3.14}$$

$$\{\widetilde{Q}, \theta_p\} = \sin^2\frac{\pi p}{J}p_{p+J/2}^i\gamma^i + \cos^2\frac{\pi p}{J}x_p^i\gamma^i\Pi + \frac{2}{a}\cos\frac{\pi p}{J}\sin^2\frac{\pi p}{J}e^{i\pi p/J}x_{p+J/2}^i\gamma^i\,. \tag{3.15}$$

In the continuum limit, focusing on the light modes with momenta close to zero, the transformations reduce to the expected expressions:

$$\begin{aligned}
&[Q, x^i] = -i\gamma^i\theta\,, && [\widetilde{Q}, x^i] = -i\gamma^i\tilde{\theta}\,, \\
&[Q, p^i] = -i\gamma^i\Pi\tilde{\theta} - 2i\gamma^i\theta'\,, && [\widetilde{Q}, p^i] = i\gamma^i\Pi\theta + 2i\gamma^i\tilde{\theta}'\,, \\
&\{Q, \theta\} = p^i\gamma^i - 2x'^i\gamma^i\,, && \{\widetilde{Q}, \theta\} = -x^i\gamma^i\Pi\,, \\
&\{Q, \tilde{\theta}\} = x^i\gamma^i\Pi\,, && \{\widetilde{Q}, \tilde{\theta}\} = p^i\gamma^i - 2x'^i\gamma^i\,.
\end{aligned} \tag{3.16}$$

A crucial observation is that there is no undesired mixing between low energy and high energy states. In particular, the $\cos^2 \pi p/J$ factor in (3.10) makes sure, for p close to $J/2$, that there is no mixing of the low energy mode $\tilde{\theta}_{p-J/2}$ into the supersymmetry transformation of the high energy mode x^i_p. This is a necessary property if we want to achieve a consistent supersymmetric truncation to the low energy sector.

4. Checking the superalgebra

The results of the previous section are very encouraging, but we also need to check that the pp-wave superalgebra emerges in the continuum limit. Straightforward calculations show that

$$\begin{aligned}
[Q, H] =& -\frac{i}{8a}\sum_{n=1}^{J}\left(x^i_{n+2} - 2x^i_n + x^i_{n-2}\right)\gamma^i\left(\theta_n - \theta_{n-1}\right) + \frac{i}{8}\sum_{n=1}^{J}\left(p^i_{n+2} - 2p^i_n + p^i_{n-2}\right)\gamma^i\theta_n - \\
&-\frac{i}{8}\sum_{n=1}^{J}\left(x^i_{n+2} - 2x^i_n + x^i_{n-2}\right)(-1)^n\gamma^i\Pi\theta_{n-1} + \frac{ia}{8}\sum_{n=1}^{J}\left(x^i_{n+2} - 2x^i_n + x^i_{n-2}\right)\gamma^i\theta_n + \\
&+\frac{ia}{8}\sum_{n=1}^{J}\left(p^i_{n+2} - 2p^i_n + p^i_{n-2}\right)(-1)^n\gamma^i\Pi\theta_n\,,
\end{aligned} \tag{4.1}$$

which can be Fourier transformed to

$$\begin{aligned}
[Q, H] =& -\frac{1}{2}\sum_{p=-J/2}^{J/2-1}\sin^2\frac{2\pi p}{J}\Big[ia(x^i_p\gamma^i\theta_{-p} + p^i_p\gamma^i\Pi\theta_{J/2-p}) + i(p^i_p\gamma^i\theta_{-p} + e^{2\pi i p/J}x^i_p\gamma^i\Pi\theta_{J/2-p}) + \\
&+\frac{2}{a}\sin\frac{\pi p}{J}e^{\pi i p/J}x^i_p\gamma^i\theta_{-p}\Big]\,.
\end{aligned} \tag{4.2}$$

Clearly, the supercharges do not commute with the hamiltonian, but it is easy to see that the commutator either vanish in the continuum limit (as $J \to \infty$), or is zero provided that we put the heavy lattice modes to zero.

We can also check that the anti-commutators of supercharges agree with the pp-wave superalgebra [1] up to terms which decouple in the continuum limit. We find:

$$\left\{Q_{\dot{a}}, Q_{\dot{b}}\right\} = 2\delta_{\dot{a}\dot{b}}(\mathcal{H} + \mathcal{P}) + 2\left(\gamma^i\Pi\gamma^j\right)_{\dot{a}\dot{b}}\left(\mathcal{R}^{ij} + \mathcal{T}^{ij}\right), \tag{4.3}$$

$$\left\{\widetilde{Q}_{\dot{a}}, \widetilde{Q}_{\dot{b}}\right\} = 2\delta_{\dot{a}\dot{b}}(\mathcal{H} - \mathcal{P}) + 2\left(\gamma^i\Pi\gamma^j\right)_{\dot{a}\dot{b}}\left(\mathcal{R}^{ij} - \mathcal{T}^{ij}\right), \tag{4.4}$$

$$\left\{Q_{(\dot{a}}, \widetilde{Q}_{\dot{b})}\right\} = 2\delta_{\dot{a}\dot{b}}\mathcal{V} + 2\left(\gamma^i\Pi\gamma^j\right)_{\dot{a}\dot{b}}\mathcal{S}^{ij}\,, \tag{4.5}$$

where

$$\mathcal{H} = \frac{a}{32}\sum_{n=1}^{J}\Big[(p_{n+2}+2p_{n+1}+p_n)^2+\frac{1}{a^2}(x_{n+3}+x_{n+2}-x_{n+1}-x_n)^2$$
$$+(x_{n+2}+2x_{n+1}+x_n)^2+\frac{2i}{a}\theta_n(5\theta_{n+1}+\theta_{n+3})-16i(-1)^n\theta_n\Pi\theta_{n+1}\Big], \quad (4.6)$$

$$\mathcal{P} = \frac{1}{32}\sum_{n=1}^{J}\left(\{p_n, x_{n+3}+3x_{n+2}+2x_{n+1}-2x_n-3x_{n-1}-x_{n-2}\}+8i\theta_n\theta_{n+2}\right), \quad (4.7)$$

$$\mathcal{V} = \frac{a}{16}\sum_{n=1}^{J}(-1)^n\Big[-p_n(p_{n+2}-p_n)-\frac{1}{a^2}x_n(x_{n+3}-3x_{n+1})+x_n(x_{n+2}-x_n)-$$
$$-\frac{i}{a}\theta_n(\theta_{n+3}-\theta_{n+1})\Big], \quad (4.8)$$

$$\mathcal{R}^{ij} = \frac{1}{16}\sum_{n=1}^{J}(-1)^n x_n^{(i}(x_{n+3}-2x_{n+2}-3x_{n+1}+2x_n)^{j)}, \quad (4.9)$$

$$\mathcal{T}^{ij} = \frac{a}{16}\sum_{n=1}^{J}(-1)^n p_n^{(i}(x_{n+2}-2x_n+x_{n-2})^{j)}, \quad (4.10)$$

$$\mathcal{S}^{ij} = \frac{1}{32}\sum_{n=1}^{J}x_n^{(i}(x_{n+3}+2x_{n+2}-x_{n+1}-4x_n-x_{n-1}+2x_{n-2}+x_{n-3})^{j)}. \quad (4.11)$$

The $\delta_{\dot{a}\dot{b}}$ terms in the anti-commutators of Q and $\widetilde{Q}$ have the right structure, the other terms are lattice artifacts and should go to zero in the continuum limit. The operator $\mathcal{H}$ is just another discretization of the string hamiltonian and $\mathcal{P}$ can be regarded as a lattice counterpart of the momentum operator. One should note, though, that $\mathcal{H}$ do *not* agree with H when evaluated on the heavy lattice modes. $\mathcal{H}$ would lead to a similar doubling problem as before, but now for the bosons, and could not serve as a new hamiltonian. This is expected, since we can only hope for the supersymmetry algebra to close when applied to the low-energy states. Indeed the difference $\mathcal{H}-H$ goes to zero in the continuum limit. The same happens to the structures that do not appear in the continuum superalgebra. The operators $\mathcal{R}^{ij}$ and $\mathcal{T}^{ij}$ contain staggered bosons, which means that they necessarily decouple when we put heavy modes to zero. The symmetric part of the anti-commutator between Q and $\widetilde{Q}$ is zero for the continuum pp-wave string.[3] The two terms in the $\{Q,\widetilde{Q}\}$ indeed vanish in the continuum limit, but for different reasons: the operator $\mathcal{V}$ is of the staggered boson type, while $\mathcal{S}^{ij}$ contains a discretized version of the second derivative of x which lacks one power of the inverse lattice spacing.

5. Conclusions

In this paper we have proposed a modification the string bit model [9] in which the fermion doubling is taken into account. By taking advantage of the doubling, we construct two

[3]The anti-symmetric part, which we do not discuss here, is proportional to a linear combination of SO(4) rotation generators [1].

fermions in the continuum from a single lattice fermion and its doubler. We have shown that the correct spectrum of the type-IIB string in the pp-wave emerges in the continuum limit. We have also checked that the supersymmetry generators obey the expected algebra if we limit ourselves to the states that remain light in the continuum limit and that supersymmetry transformations do not mix the light states with heavy lattice modes. Our approach crucially depends on the fact that world-sheet fermions of the IIB string belong to the same representation of SO(8). It would be interesting to see whether a similar construction can be made to work for the IIA string where the two sets of fermions transform differently under SO(8).

We have not discussed multi-string states and have not addressed the problem of constructing interaction vertices. We believe, however, that it should be relatively straightforward to make the appropriate modifications of the original string bit model [9] given our results about fermion doubling.

One of the main motivations for considering string bit models is the hope that they may provide an adequate dual description of the SYM theory [9, 20]. In the context of the pp-wave strings, the SYM operators that are dual to fermion states of the string are known and were shown to combine into complete supermultiplets [21], even before taking the continuum limit, which shows that supersymmetry is really important here. It is also worth mentioning that discretized analogs of the string hamiltonian which naturally arise in the SYM theory are hamiltonians of integrable spin chains [22, 23, 24]. It would be very interesting to find connections between the integrable spin chains and the string bit models.

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